

Six Sigma Rolled Throughput Yield

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Scenario:

You are touring a production process and you see numerous steps with inspection followed by scrap or rework.

At several such steps, you ask, “What is the yield?”

Answer at step 1: **95%**.

Answer at step 2: **90%**.

...

Answer at final test: **99%**.

Then you ask, “What is the overall yield?”

Answer: **99%**

Repeat the question:

You finally ask,

**“What percent of product
goes straight through the process
without problems?”**

Answer: _____

If only 10% of units go straight through the process without being scrapped or reworked, then 90% of units cost more than they should and may deliver less value to customers than the “straight through” 10%

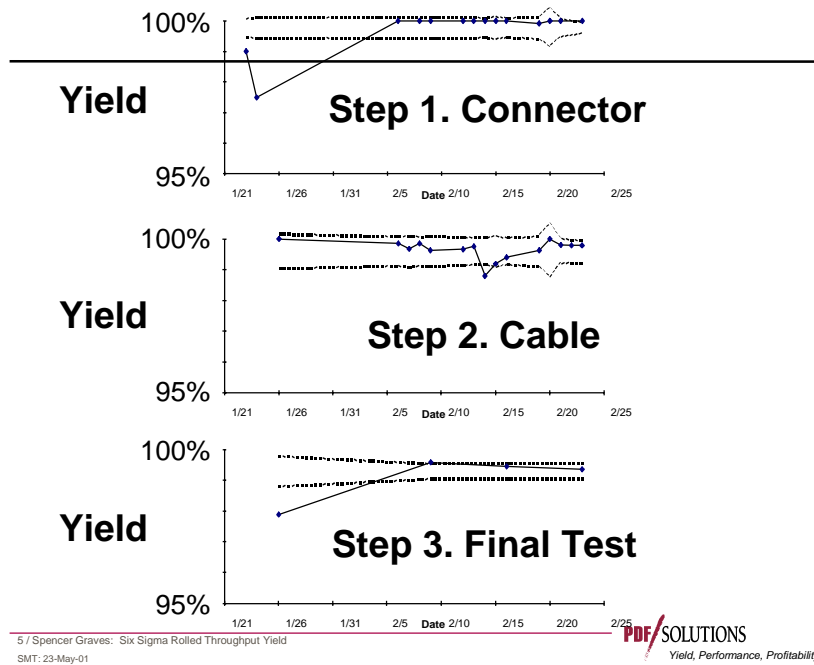
How to Evaluate the “Goodness” or Level of Problems to Allocate \$ for Improvement?

One easy measure:

Rolled Throughput Yield (RTY):

$$p = p_1 p_2 \cdots p_n = \prod_{i=1}^n p_i$$

p_i = yield at step i



January 26

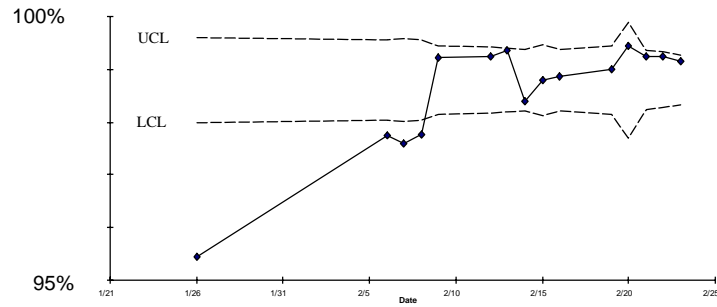
$$p_2 = 100\%$$

$$p_1 = 97.5\% \text{ from January 24}$$

$$p_3 = 97.9\% \text{ for the week}$$

$$p = p_1 p_2 p_3 = 95.4\%$$

Rolled Throughput Yield



Rolled Throughput Yield
= product of the yields at all steps in the process

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We can compute a number

- Does it have a purpose?

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Net Yield in Chemical Engineering

**The percent of the primary input
that is converted to output.**

■ This

- selects one p_i as most important,
- ignores others
- is appropriate in some cases
- is NOT sensible generally

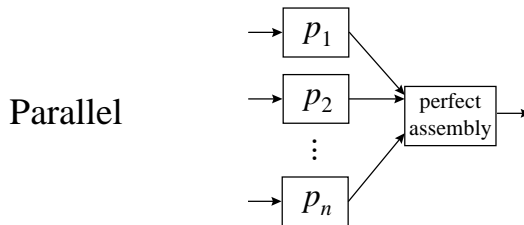
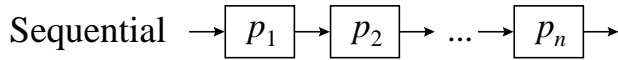
Interpretation:

Japanese: *A Dictionary of Total Quality Control Terms*

“non-adjusted rate,” “go-through rate”:

Percent of product that passes straight through
the production process
without delays due to quality problems.

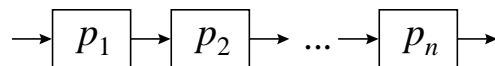
Rework (no scrap):



If the probability of problems at one step is *independent* of problems at other steps, then

RTY = percent of product without problems

Scrap in a Sequential Process



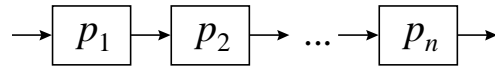
The percent of product that finishes step i

= p_i times the percent of product that finished step $(i - 1)$.

Thus,

RTY = percent of product entering the process that actually finishes.

Scrap in a Sequential Process

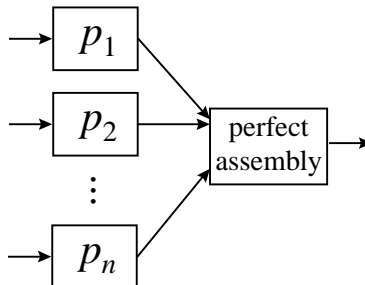


p_2 = percent of product that finishes step 2
given it was good leaving step 1
So, the percent of product starts that finish step 2
 $= p_2 p_1$

Thus,

RTY = percent of product entering the process
that actually finishes

Scrap in a Parallel Process



What is the “right” number for “yield”?

... it depends

A Reasonable Index

- **For a process with scrap in parallel steps, the product of the yields is**
 - a *useful* index of process quality,
 - without an interpretation as a % of some obvious count of units

Scrap and Rework

- **Production problems**
 - Increase labor and material costs
 - May cause scheduling and inventory difficulties
 - May contribute to reliability problems
- **Scrap and rework**
 - The cost of scrap may be more obvious and more easily quantified
 - Many costs associated with rework may be more subtle and harder to quantify
- **For simplicity, we ignore the distinction between scrap and rework**

Process Changes?

■ Add inspections

- If you catch EARLIER problems that would have been caught later: RTY is essentially unchanged
- If you catch problems previously overlooked: the new RTY is a better reflection of process health than without the added inspections

■ If you change the way you think about the process

- You don't need to discard RTY history

Defects and Defectives

■ Harry and Schroeder (2000) and Breyfogle (1999) use the Poisson assumption to translate between

- the proportion of units with defects (p), and
- defects per unit (DPU, u)

$$p = \exp(-u)$$
$$u = [-\ln(p)]$$

Breyfogle, F. W., III (1999) *Implementing Six Sigma* (NY: Wiley)

Harry, M., and Schroeder, R. (2000) *Six Sigma* (NY: Currency)

Rolled Throughput Yield for a General Process

$$p = \prod_{i=1}^n p_i = \text{a reasonable index of delays}$$

and productivity losses in the process.

■ Use this for

- scrap and rework
- series or parallel steps
- defects or defectives

■ Translate freely, if approximately, to DPU via

$$u = [-\ln(p)] = \sum_{j=1}^n u_j$$

Rolled Throughput Yield

■ Defined this way, RTY

- is simple, easy to remember, easy to compute
- provides a reasonable first cut analysis in many cases

■ In many cases, RTY provides an obvious answer that would not be obvious without it

■ In other cases, a Pareto using RTY may eliminate some options, narrowing the scope for more careful studies of potential problem areas

RTY in Process Control

■ We discuss here

- Variability
- Statistical control limits
- Example

Process Variability

$\hat{p}_{j,t}$ = *observed* yield at step j on day t .

p_j = the *average* yield at step j assuming the process is stable,

\hat{p}_t = the *observed* Rolled Throughput Yield for day t

$$\hat{p}_t = \prod_j^n \hat{p}_{j,t}$$

or

$$\hat{u}_t = \sum_j^n \hat{u}_{j,t}$$

$$\text{var}(\hat{u}_{j,t}) = \begin{cases} u_j/n_{j,t} & \text{if counting defects} \\ (1-p_j)/(p_j n_{j,t}) & \text{if counting defectives} \\ \text{var}(\hat{u}_{j,t}) & \text{with overdispersion} \end{cases}$$

Statistical Control Limits

$$\text{var}(\hat{u}_{j,t}) = \begin{cases} u_j / n_{j,t} & \text{if counting defects} \\ (1 - p_j) / (p_j n_{j,t}) & \text{if counting defectives} \\ \text{var}(\hat{u}_{j,t}) & \text{with overdispersion} \end{cases}$$

$$\text{var}(\hat{u}_t) = \sum_j^n \text{var}(\hat{u}_{j,t})$$

$$\text{UCL}(\hat{u}_t) = u + 3\sqrt{\text{var}(\hat{u}_t)}$$

$$\text{LCL}(\hat{u}_t) = u - 3\sqrt{\text{var}(\hat{u}_t)}$$

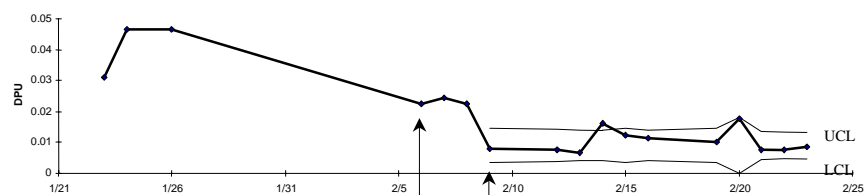
$$\text{UCL}(\hat{p}_t) = \exp[-\text{LCL}(\hat{u}_t)]$$

$$\text{LCL}(\hat{p}_t) = \exp[-\text{UCL}(\hat{u}_t)]$$

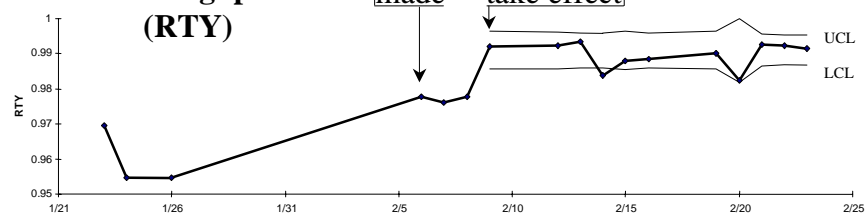
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Defects per Unit (DPU)



Rolled Throughput Yield (RTY)



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A Forecasted Pareto

[1]	[2]	[3]	[4]
Product	Rolled Throughput Yield (last 3 months)	Sales Forecast for next year (K\$)	Estimated Annual Savings = $0.2 \times$ $(1 - [2]) \times [3]$ (K\$)
A	0.999	9,000	2
B	0.77	3,300	152
C	0.60	640	51
D	0.997	1,400	1

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Evaluation

[1]	[2]	[3]	[4]
Product	Rolled Throughput Yield (last 3 months)	Sales Forecast for next year (K\$)	Estimated Annual Savings = $0.2 \times$ $(1 - [2]) \times [3]$ (K\$)
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C	0.60	640	51
D	0.997	1,400	1

- This analysis suggests that product B problems may merit some attention
- A brief review of product C may also be appropriate

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Conclusions

Rolled throughput yield

■ or the complementary DPU Poisson rate

■ provides a simple way to summarize

- defects and defectives
- scrap and rework
- series and parallel steps
- for process control and project selection

Spencer Graves (1998) “Statistical Quality Control of a Multistep Production Process using Total Process Yield”,

Quality Engineering, 11(2), pp. 187-195

_____ (2001) “Six Sigma Rolled Throughput Yield”,

Quality Engineering, 14(2) (forthcoming)

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